

# On the estimation of variance components in Gage R&R studies using Ranges & ANOVA

B. Ramesh, K.V.S. Sarma

**Abstract**— This paper discusses various methods of estimating the variance components in a measurement system and makes a comparison of the method of ranges with that of Analysis of Variance (ANOVA). We have developed Excel templates to perform calculations and it is shown that the ratios of variance components to the total variance, add up to unity in the case of ANOVA and in the modified range method. The procedures are numerically illustrated. The effect of changes in the process mean and standard deviation on the vital ratios is studied by simulation.

**Keywords**— AIAG, Gage R&R, Modified AIAG, Modified Vital Ratio, Nested ANOVA, r-coefficient, Vital Ratio

## 1 INTRODUCTION

THE determination of an outcome of a quality characteristic is called measurement. It is a knowledge which evaluates the unknown quality in-terms of numerical values. These measurement systems are used every day in manufacturing, research, development, sales and marketing. Measurement System Analysis (MSA) is designed to help engineers, quality professionals in assessing, monitoring and reducing the variation that includes features of a measurement system like linearity, stability, repeatability, reproducibility (Gage R&R) and the calibration of measurement equipment. Thus it is a vital component for many quality improvement initiatives. The Automotive Industry Action Group (2006) has prepared a manual to work out the analysis of a measurement system. The basic idea is to estimate the variation in the measured values that can be attributed to several factors like operator, equipment etc. The method ranges is one approach to estimate the variance components while Analysis of Variance (ANOVA) with random effects model, is another way.

The process of analyzing any measurement system involves three key dimensions viz., operator, equipment and material.

The causes of variation due to these three key dimensions should be kept at minimum so that the efficiency of the measurement system is maximized. In this paper we compare three existing methods of estimating the variance components viz., i) AIAG method ii) ANOVA method and iii) Modified AIAG method.

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## 2 REVIEW OF MSA CONCEPTS

The complete MSA is carried out by studying the measurement system variation to understand the components of variation. Let  $X$  denote the measurement made on a part using the given measurement system. It is common to assume that  $X \sim N(\mu, \sigma^2)$  and this assumption holds good in most cases of large scale production.

Variation in measurements occur over a period of time or when other assignable causes like material changes, operator changes and changes in machine settings take place. The variation can be classified into a) changes in the location (mean value) of the process and b) changes in width (variation) of the process. These two concepts are elaborated below.

### 2.1 Location variation

The changes that occur in the process average accounts for deviations in the quality. The following are some indices of location variation.

- a) Accuracy (or) Bias - It is a measure of the distance (closeness of agreement) between the average value of a large number of observed values of the characteristic and the true value or reference value. The reference value is obtained by a standardized procedure with properly calibrated equipment.
- b) Stability - It is the consistency of the performance over time and indicates the absence of assignable causes of variation, leaving only random variation. It measures the change in bias over time. Stability refers to the difference in the averages of at least two sets of measurements obtained with the same gage on the same characteristics taken at different times.
- c) Linearity - It measures the consistency of accuracy over the range of measurements. When the true value is high, the observed value also should be high and vice versa. Linearity is expressed as,  $|\text{Slope}| * \text{Process variation}$ , where  $||$  indicates absolute value.

## 2.2 Width variation

The total variation found in the measurement system can be divided into the following components.

- a) Precision (closeness) - Precision is the standard deviation of the measurement system. The smaller the spread of the distribution, the better is the precision. Precision can be separated into two components, called repeatability and reproducibility.
- b) Repeatability: Repeatability is the inherent variation within the measuring instrument and is represented by  $\sigma_{\text{repeatability}}$ . It is the variation due to measurement equipment obtained with one instrument used several times by one appraiser while measuring the parts. This is also known as Equipment variation (EV).
- c) Reproducibility: Reproducibility is the variation due to differences in appraisers denoted by  $\sigma_{\text{reproducibility}}$ . It is the variation in the average of measurements made by different operators using the same equipment when measuring the same characteristics on the same part. This is also known as Appraiser variation (AV).
- d) Gage R&R - Gage Repeatability and Reproducibility (R&R) is a measure which represents the variation due to the measurement system as a whole. It determines how much of the observed process variation has occurred due to the measurement system variation. It is the combined estimate of R&R and denoted by

$$\sigma_{\text{Gauge}}^2 + \sigma_{\text{Repeatability}}^2 + \sigma_{\text{Reproducibility}}^2$$

The problem is to estimate the variance components either by generating data through an experiment or by collecting data from the production line using the measurement system.

## 3 THE USE OF EXPERIMENTAL DESIGN

Any item from the process on which a measurement is made by the operator with given equipment is called a part. The total variation in the measurement could be due to part, due to equipment or due to operator. These three components shall be estimated by observing the process over a period of time or by conducting an experiment. Factorial experiments with random effects model are used to estimate the variance components. Let the system contains two factors i) Operator (A) and ii) Part (B) which contribute to variation, apart from random variation. Then the variation due to A and variation due to B within A, denoted by B(A) represents  $\sigma_{\text{reproducibility}}$  and the variation due to the experimental error (residual) denotes  $\sigma_{\text{repeatability}}$ .

Let  $Y_{ijk}$  denote the  $k^{\text{th}}$  value obtained on the  $j^{\text{th}}$  part by the  $i^{\text{th}}$  operator. Then the design for this experiment can be expressed by the two way random effects model given by,

$$Y_{ijk} = \mu + A_i + B(A)_{j(i)} + \varepsilon_{ijk} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \\ k = 1, 2, \dots, n \end{cases} \quad (1)$$

Where a = number of operators, b= number of parts, n= number of replications and B(A) denotes the parts within operator.

If  $\sigma_Y^2$  denotes the total variance in the experimental data then it can be split into components as follows.

$$(\sigma_Y^2) = (\sigma_A^2) + (\sigma_{B(A)}^2) + (\sigma_\varepsilon^2) \quad (2)$$

$\sigma_\varepsilon^2$  is called  $\sigma^2$  repeatability or Equipment variation (EV).

We have considered three methods to estimate the variance components and are as follows.

## 4 METHODS OF ESTIMATING VARIANCE COMPONENTS

There are three methods in use, for estimating the variance components in a measurement system. In addition to these, there are three ratios used to evaluate the performance of the measurement system. We call them vital ratios discussed in detail below.

### 4.1 Variance components by AIAG method ( $\bar{R}$ – method)

This method depends on the ranges and the bias factor  $d_2$  to estimate the variance components. It is known that for a normally distributed data, the ratio  $\left[ \frac{\bar{R}}{d_2} \right]$  is an estimate of the process standard deviation  $\sigma$ , where  $\bar{R}$  is the mean of ranges in the subgroups. For different sizes of the sub groups the values of  $d_2$  are tabulated. Another related constant is  $d_2^*$  which is a function of the levels of the factor (like operator) in the measurement system. The value of both  $d_2$  and  $d_2^*$  for different sample sizes were provided by Duncan (1955). For  $n \geq 25$ , the constant  $d_2$  shall be used and for  $n < 25$  the constant  $d_2^*$  shall be used. The constant for the factor equipment is denoted by  $d_{2,e}^*$  and for parts it is  $d_{2,p}^*$ .

We consider a measurement system with a operators (appraisers) and b parts and n replications (trials).

The  $\bar{R}$  method makes use of means of sample ranges  $\bar{R}$  to estimate  $\sigma_e$  as  $\hat{\sigma}_e = \frac{\bar{R}}{d_2}$  where  $\bar{R} = \frac{\sum_{i=1}^a R_i}{a}$  and

$$R_i = \text{Max}(X_{ij}) - \text{Min}(X_{ij}) \text{ for the } i^{\text{th}} \text{ sample for } j=1,2,\dots,b.$$

From the tables of  $d_2$  values we get  $d_{2,a}^* = 1.41$  and  $1.91$  for  $a = 2$  and  $a = 3$  respectively.

### Illustration-1

The sample lay out with  $a = 3$ ,  $b = 5$  and  $n = 2$  appears as shown in Table-1 where X denotes the actual measurement made on each part, in suitable units.

Table-1: Sample data from an experiment on the weight of a gasket X (in grams)

Operator - 1				Operator - 2				Operator - 2			
Replicate	Part	X	Range within part	Replicate	Part	X	Range within part	Replicate	Part	X	Range within part
1	1	167	5	1	1	155	2	1	1	152	3
1	2	210	3	1	2	206	7	1	2	206	3
1	3	187	4	1	3	182	3	1	3	180	1
1	4	189	7	1	4	184	6	1	4	180	2
1	5	156	9	1	5	143	1	1	5	146	8
2	1	162	--	2	1	157	--	2	1	155	--
2	2	213	--	2	2	199	--	2	2	203	--
2	3	183	--	2	3	179	--	2	3	181	--
2	4	196	--	2	4	178	--	2	4	182	--
2	5	147	--	2	5	142	--	2	5	154	--

Since each part is measured twice, the ranges within each part are calculated for each operator and the average of these ranges is found to give  $\bar{R} = 4.267$ . Since all the 30 data values are treated as a single group we have  $n > 25$  and hence  $d_2 = 1.128$ .

Wheeler (2009) has provided the following estimators.

$$1. \text{ Equipment Variation (EV)} = \hat{\sigma}_{\text{Repeatability}} = \hat{\sigma}_e = \frac{\bar{R}}{d_2}$$

$$2. \text{ Appraiser Variation (AV)} = \hat{\sigma}_{\text{Reproducibility}} = \hat{\sigma}_a = \sqrt{\left[ \frac{R_a}{d_{2,a}^*} \right]^2 - \left( \left[ \frac{a}{a.b.n} \right] [\hat{\sigma}_e]^2 \right)}$$

Where  $R_a$  is the range of operator averages. For some data values, the content inside the square root turns out to be negative in which case it is reset to zero.

$$3. \text{ Gage R\&R (GR\&R)} = \hat{\sigma}_{\text{R\&R}} = \sqrt{(\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2}$$

$$4. \text{ Part Variation (PV)} = \hat{\sigma}_p = \sqrt{\left[ \frac{R_p}{d_{2,p}^*} \right]^2} \quad \text{where } R_p \text{ is}$$

the range of the parts.

$$5. \text{ Total Variation (TV)} = \hat{\sigma}_T = \sqrt{(\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2 + (\hat{\sigma}_p)^2} = \sqrt{(\hat{\sigma}_{\text{R\&R}})^2 + (\hat{\sigma}_p)^2}$$

After estimating these variance components, we express each of them as a percentage of TV. We get three Vital Ratios (VR). Each VR represents the portion of the total variation attributable to the factor under consideration. These ratios can be expressed as percentages. The AIAG group has used the ratios as EV/TV, AV/TV, PV/TV and GRR/TV for the equipment, operator, product and R&R respectively.

Wheeler (2009) observed that these VRs are not ratios in a proper sense as they would not add to unity and proposed the following modified Vital Ratios which can be interpreted in a nice way.

- a)  $VR_{\text{repeat}} = EV^2/TV^2$  in place of EV/TV
- b)  $VR_{\text{reprod}} = AV^2/TV^2$  in place of AV/TV
- c)  $VR_{\text{part}} = PV^2/TV^2$  in place of PV/TV
- d)  $VR_{\text{GRR}} = GRR^2/TV^2$  in place of GRR/TV

Ermer (2006) observed that the use of ratio of variances in place of the ratio of standard deviations is justified due to the fact that

$[\sigma_{a+b}^2 = \sigma_a^2 + \sigma_b^2]$  but  $[\sigma_{a+b} \neq \sigma_a + \sigma_b]$  and the same can be established by the Pythagorean theorem. Wheeler (2006) also showed that the VR method based on the standard deviations leads to trigonometrically derived quantities which do not add to unity. However it can be seen that  $(VR_{\text{repeat}} + VR_{\text{reprod}} + VR_{\text{part}}) = (VR_{\text{GRR}} + VR_{\text{part}}) = 1$ . For the data given in illustration-1 we get the results as shown in Table-2.

Table-2: Variance components and variance ratios for the data in illustration-1

Component	Source of variation				
	Equipment	AV	Gage R&R	PV	TV
Variance	3.783	4.287	5.717	23.45	24.14
Variance Ratio	0.157	0.178	0.334	0.972	1.306
modified Variance Ratios (EV <sup>2</sup> /TV <sup>2</sup> )	0.025	0.031	0.056	0.944	1.000

It can be seen that the ratios in the fourth row of Table-2 do not add up to unity while the modified ratios in the fifth row add up to unity.

We have got  $VR_{\text{part}} = 0.944$  which means that 94.4% of the variation in the measurements can be attributed to parts (production) while the equipment variation accounts for 2.5% and the appraiser variation consumes 3.1%. The sum of these ratios adds up to unity.

Now we consider another method called  $\overline{WR}$  method to estimate the variance components by using within sample ranges.

#### 4.2 Variance components by modified AIAG

##### $\overline{WR}$ – method)

In this method  $\hat{\sigma}_{\text{Repeatability}}$  is estimated by taking the average of the ranges within parts instead of taking the average range from the entire data. For each operator, the average of the ranges within parts is calculated. The average of these averages is used to find the EV. It also makes use of the constant  $d_2^*$  whose values are available in Duncan (1955).

The variance components in this case are as follows.

1.  $\hat{\sigma}_e = \frac{\overline{WR}}{d_{2,e}^*}$  where  $\overline{WR}$  is the average of the averages of range within parts.

2.  $\hat{\sigma}_a = \sqrt{\left[\frac{R_a}{d_{2,a}^*}\right]^2 - \left(\frac{[\hat{\sigma}_e]^2}{bn}\right)}$

3.  $\hat{\sigma}_{R\&R} = \sqrt{(\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2}$

4.  $\hat{\sigma}_p = \sqrt{\left[\frac{R_p}{d_{2,p}^*}\right]^2 - \left(\frac{[\hat{\sigma}_e]^2}{an}\right)}$

5.  $\hat{\sigma}_T = \sqrt{(\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2 + (\hat{\sigma}_p)^2} = \sqrt{(\hat{\sigma}_{R\&R})^2 + (\hat{\sigma}_p)^2}$

For some data values, the content inside the square root turns out to be negative in which case AV and PV are set to zero.

For the case of EV, the numerator is based on  $m = 5 \times 3 = 15$  ranges ( $< 30$ ) and with  $n = 2$  replications we get  $d_{2,e}^* = 1.15$ .

Similarly we get  $d_{2,a}^* = 1.91$  for AV ( $a = 3$ ) and  $d_{2,p}^* = 2.48$  for PV ( $b = 5$ ).

**Illustration-2**

Reconsider the data in Table-1 of illustration-1. The method of using within ranges of parts produces the following intermediate calculations.

Table-3: Data with intermediate calculations for WR method.												
Operator - 1				Operator - 2				Operator - 2				
Replicate	Part	X	Range within part	Replicate	Part	X	Range within part	Replicate	Part	X	Range within part	
1	1	167	5	1	1	155	2	1	1	152	3	
1	2	210	3	1	2	206	7	1	2	206	3	
1	3	187	4	1	3	182	3	1	3	180	1	
1	4	189	7	1	4	184	6	1	4	180	2	
1	5	156	9	1	5	143	1	1	5	146	8	
2	1	162	--	2	1	157	--	2	1	155	--	
2	2	213	--	2	2	199	--	2	2	203	--	
2	3	183	--	2	3	179	--	2	3	181	--	
2	4	196	--	2	4	178	--	2	4	182	--	
2	5	147	--	2	5	142	--	2	5	154	--	
WR = 5.6				WR = 3.8				WR = 3.4				
Operator Average= 181.				Operator Average= 172.5				Operator Average= 173.9				

From the Table-3 we get the following intermediate calculations.

- a) The average of WR gives  $\overline{WR} = 4.267$
- b) The range of operator averages becomes 8.5
- c) Each part appears twice with each operator and hence the average of X is based on 6 observations and hence we get five values of part averages as {158.0, 206.2, 182.0, 184.8, 148.0} so that the range of these averages is 58.167.

The summary of results including the modified VRs is shown in Table-4.

Component	Source of variation				
	Equipment	Appraiser	Parts	Part	Total
Variance	13.765	18.43	32.19	548.96	581.29
Modified Vital Ratios	0.024	0.032	0.055	0.944	1.000

It can be seen that the sum of modified VR's is unity. Closer this sum to unity better is the method estimating the variance components.

In a normally distributed data the range method, though unbiased, is not the only way of estimating the variance components. The Analysis of Variance (ANOVA) with random effects model is a known procedure. In the following section, we consider Nested ANOVA method and compare the estimate by the three methods.

**4.3 Variance components by Nested ANOVA method**

According to Montgomery (2002) , in an experiment if the levels of one factor are similar but not identical for different levels of another factor then such an arrangement is said to be nested design. Here we have the levels of factor B nested under the levels of factor A. There are b parts of raw materials available from each operator (a), and n trials are to be taken for each part. This is a two-stage nested design, with parts nested within operators.

We use Two-way Nested ANOVA with random effects model to estimate the variance components.

Define

- $y_{i..}$  = mean of the  $i^{th}$  operator
- $y_{.j.}$  = mean of the  $j^{th}$  part
- $y_{...}$  = overall mean of all observations.
- $y_{ij.}$  = mean of observations at the  $i^{th}$  operator and  $j^{th}$  part.

Then

- 1. Sum of squares (SS) due to operator ( $SS_A$ ) =  $\sum_{i=1}^a \left(\frac{y_{i..}^2}{bn}\right) - \left(\frac{y_{...}^2}{abn}\right)$
- 2. SS due to parts within operator within A ( $SS_{B(A)}$ ) =  $\sum_{i=1}^a \sum_{j=1}^b \left(\frac{y_{ij.}^2}{n}\right) - \sum_{i=1}^a \left(\frac{y_{i..}^2}{bn}\right)$
- 3. SS due to Residual ( $SS_e$ ) =  $\left[\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n (y_{ijk}^2)\right] - \left[\sum_{i=1}^a \sum_{j=1}^b \left(\frac{y_{ij.}^2}{n}\right)\right]$
- 4. Total SS ( $SS_T$ ) =  $\left[\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n (y_{ijk}^2)\right] - \left(\frac{y_{...}^2}{abn}\right)$

The corresponding mean sum of squares (MSS) are obtained by dividing each SS by the corresponding degrees of freedom.



The variance components are given by

1.  $\hat{\sigma}_e^2 = [MSS_\epsilon]$
2.  $\hat{\sigma}_a^2 = \left[ \frac{MSS_A - MSS_{B(A)}}{na} \right]$
3.  $\hat{\sigma}_p^2 = \left[ \frac{MSS_{B(A)} - MSS_\epsilon}{n} \right]$
4.  $\hat{\sigma}_{R\&R}^2 = (\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2$
5.  $\hat{\sigma}_T^2 = (\hat{\sigma}_e)^2 + (\hat{\sigma}_a)^2 + (\hat{\sigma}_p)^2 = (\hat{\sigma}_{R\&R})^2 + (\hat{\sigma}_p)^2$

**Illustration-3**

Reconsider the data given in illustration-1. By using Nested-ANOVA method the following output is found.

Component	Source of variation				
	Equipment	Appraiser	GageR&R	Part	Total
Variance	12.200	0.000	12.200	531.167	543.367
Modified Vital Ratios	0.022	0.000	0.022	0.978	1.000

The VRs are found in the usual way for which the ratio property

holds well. It can be seen that  $\left( \frac{\hat{\sigma}_e^2 + \hat{\sigma}_a^2 + \hat{\sigma}_p^2}{\hat{\sigma}_T^2} \right) = 1$  which

means that the relative utilities add up to unity which describes that how strongly the units in the same group resemble each other and is used to study the performance of a measurement system.

Remark: It is possible that for certain data sets a vital ratio turns out to be more than unity, in which case it is reset to unity.

**4.4 Illustrative comparison of the three methods**

We considered a = 3, b = 5 and n = 2 and generated a random sample from N(μ,σ<sup>2</sup>) by using Data Analysis Pak of Excel with given mean (μ) & Standard Deviation (σ). For μ = 175 & σ = 2.5 the variance components and the vital ratios are shown in Table-6.

Variance Component	$\bar{R}$ - method	ANOVA method	$\overline{WR}$ - method
Repeatability (EV)	14.31 (2.45%)	12.20 (2.25%)	13.77 (2.37%)
Reproducibility (AV)	18.37 (3.15%)	0.00 (0.00%)	18.43 (3.17%)
Combined gage R&R (GRR)	32.68 (5.61%)	12.20 (2.25%)	32.19 (5.54%)
Product Variation (PV)	550.10 (94.40%)	531.17 (97.75%)	548.96 (94.44%)
Total Variation (TV)	582.79	543.37	581.29
Sum of percentage contributions	100.00%	100.00%	100.00%

It follows that both ANOVA and  $\overline{WR}$  methods produce meaningful percentage contributions of different components to the measurement system variation. The method of using ratio of

standard deviations is not a correct method, though the percentages individually appear to be correct.

**4.5 Performance indicators**

Minitab provides a module for Gage R & R studies using nested ANOVA. We have developed a template in Excel to work out the calculations for a = 3, b = 5 and n = 2 as shown in Figure-1. In the Minitab output the Gage R&R section produces variance components and % contribution of each component to total variance. These are nothing but the VRs defined in section 4.1.

A single index of measurement system performance is the

r - coefficient given by,

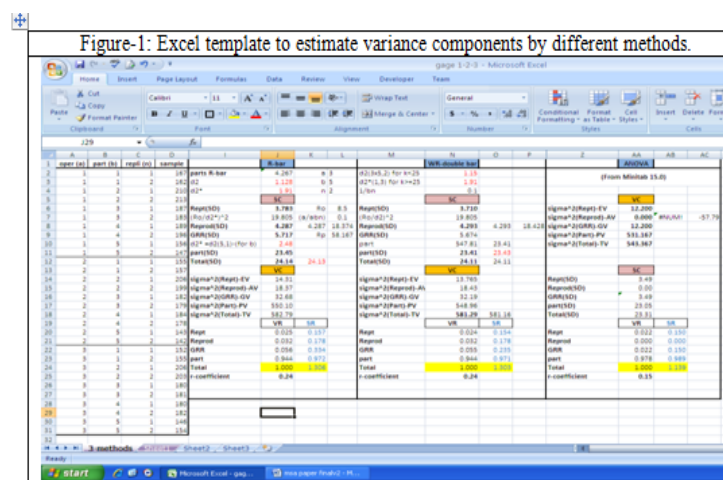
$$r = \left[ \frac{\left( \sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_a^2} \right)}{\left( \sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_a^2 + \hat{\sigma}_p^2} \right)} \right]$$

Table-7 shows the convention used to classify a process into one of the three categories.

r ≤ 0.1	Acceptable
0.1 < r ≤ 0.3	Marginal
0.3 < r	Unacceptable

The value of r is 0.24 by the  $\bar{R}$  - method and  $\overline{WR}$  - method but it is only 0.15 by ANOVA. However by all the three methods the process is classified as marginal variation in the measurement system.

The computation of above mentioned three methods is shown in Figure-1.



**5 CONCLUSIONS**

From the above study we observe the following.

- 1) More variation is due to EV when compared with AV & PV.
- 2) The Equipment Variation needed improvement.

- 3) ANOVA gives the exact ratio than  $\bar{R}$  and  $\overline{WR}$  method.
- 4) By posting the experimental data into the column-D of the Excel template, the calculations are automatically changed.

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